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$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-1} \frac{dy^2}{dx^2} = g.$$

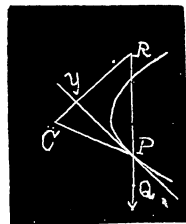
This is the differential equation of the catenary of uniform strength.
See Routh's Statics, page 329, or Minchin's Statics, page 315.

64. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Find the caustic by reflection of an hyperbola, the bright point being the center.

Solution by the PROPOSER.

Let RPQ be a reflected ray, Q a point of the caustic where RPQ touches the caustic, R the corresponding point on the secondary caustic. Then RQ is the radius of curvature of the secondary caustic, that is, $RQ=2\rho$, where ρ is the radius of curvature of the locus of y , the foot of the perpendicular from the center C on the tangent at P . Let δ be the perpendicular from C on the tangent at y to the first pedal, then $\delta r = p^2$ (Williamson's Differential Calculus, Art. 188, page 228, Sixth Edition).



Differentiating, $r \frac{d\delta}{dp} + \delta \frac{dr}{dp} = 2p$. But $\rho = \frac{pdp}{d\delta}$; and since $p^2(r^2 - a^2 + b^2) = a^2b^2$ from the hyperbola.

$$\therefore \frac{rdr}{dp} = -\frac{a^2b^2}{p^3}. \quad \therefore \frac{r}{\rho} - \frac{p^2}{r} \cdot \frac{a^2b^2}{p^4r} = 2, \text{ or } \frac{a^2b^2}{p^2r^2} = \frac{r}{\rho} - 2.$$

Let (m, n) be the coördinates of R , ψ the eccentric angle of P , (x, y) the coördinates of Q . Then

$$\frac{QR}{QP} = \frac{x-m}{x-a\sec\psi} = \frac{y+n}{y-b\tan\psi} = \frac{2\rho}{2\rho-r}.$$

In triangle CPR , $Cy=Ry$, $CP=RP=r$.

$$\therefore \frac{x-m}{x-a\sec\psi} = \frac{y+n}{y-b\tan\psi} = -\frac{2p^2r^2}{a^2b^2} = \frac{2r^2}{a^2-r^2-b^2}.$$

$$\therefore x(a^2-b^2-3r^2) = m(a^2-b^2-r^2) - 2r^2a\sec\psi.$$

$$y(a^2-b^2-3r^2) + n(a^2-b^2-r^2) + 2r^2b\tan\psi = 0.$$

$$\text{Now } m^2 + n^2 = 4p^2, \quad (m-a\sec\psi)^2 + (n+b\tan\psi)^2 = r^2.$$

$$\therefore m = \frac{2ab^2\sec\psi}{a^2\tan^2\psi + b^2\sec^2\psi}, \quad n = \frac{2a^2b\tan\psi}{a^2\tan^2\psi + b^2\sec^2\psi}.$$

$$\text{But } m(a^2-b^2-r^2) = -m(r^2-a^2+b^2) = -m(a^2\tan^2\psi + b^2\sec^2\psi) = -2ab^2\sec\psi.$$

$$\therefore x(a^2 - b^2 - 3r^2) = -2a \sec \psi (r^2 + b^2) = -2a \sec^3 \psi (a^2 + b^2).$$

$$y(a^2 - b^2 - 3r^2) = -2b \tan \psi (r^2 - a^2) = -2b \tan^3 \psi (a^2 + b^2).$$

By division, $\sin \psi = (y/b)^{\frac{1}{3}} / (x/a)^{\frac{1}{3}}$.

Eliminating ψ between the last two equations,

$$[(x/a)^{\frac{2}{3}} - (y/b)^{\frac{2}{3}}](a^2 - b^2 - 3r^2)^{\frac{2}{3}} = [2(a^2 + b^2)]^{\frac{2}{3}}.$$

$$\therefore [(x/a)^{\frac{2}{3}} - (y/b)^{\frac{2}{3}}]^{\frac{3}{2}}(a^2 - b^2 - 3r^2) = 2(a^2 + b^2).$$

$$\text{Now } r^2 = a^2 \sec^2 \psi + b^2 \tan^2 \psi = \frac{a^2(x/a)^{\frac{2}{3}} + b^2(y/b)^{\frac{2}{3}}}{(x/a)^{\frac{2}{3}} - (y/b)^{\frac{2}{3}}}.$$

$$\therefore [(x/a)^{\frac{2}{3}} - (y/b)^{\frac{2}{3}}]^{\frac{3}{2}} \{ (a^2 - b^2) [(x/a)^{\frac{2}{3}} - (y/b)^{\frac{2}{3}}] - 3[a^2(x/a)^{\frac{2}{3}} + b^2(y/b)^{\frac{2}{3}}] \},$$

$$= 2(a^2 + b^2).$$

or $[(x/a)^{\frac{2}{3}} - (y/b)^{\frac{2}{3}}]^{\frac{3}{2}} [(y/b)^{\frac{2}{3}} (\frac{1}{2}a^2 - b^2) - (x/a)^{\frac{2}{3}} (a^2 + \frac{1}{2}b^2)] = a^2 + b^2$, the caustic required.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

104. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

If I should buy goods at a price 20% higher than I did buy them, and sell the goods for the same amount that I did sell them, I would gain 25% less than I did gain. What per cent. did I gain? (Solve by Arithmetic).

105. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

A teacher looks at his watch when leaving school at noon. When he comes back he finds that the hour hand and the minute hand had just changed places (that they had when he left the school). What time was it when he left, and what time when he came back to school? (Solve by Arithmetic).

*** Solutions of these problems should be sent to B. F. Finkel not later than February 10.

GEOMETRY.

111. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Given that the area of a triangle is equal to half the product of two sides and the sine of the included angle, prove that $\sin(x+y) = \sin x \cos y + \cos x \sin y$.

112. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

The tangent planes at A, B, C, D to the sphere circumscribing the tetrahedron $ABCD$ form a tetrahedron $abcd$; prove that Aa, Bb, Cc, Dd will meet in a point if $BC \cdot AD = CA \cdot BD = AB \cdot CD$.